

# INCLUSIVE CLASSROOM EXPERIENCES IN NEW YORK CITY

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## Abstract

*This study offers a window into early childhood and elementary school inclusive mathematics classrooms, which were held in New York City.*

## 1. Introduction:

The key is a well-prepared class. We open a window to the New York Staten Island public schools' early childhood and elementary classrooms where co-teacher candidates use well-designed activities. These activities were developed by the Freudenthal Institute in the Netherlands. These series of activities have the built-in constraint to develop number sense and to understand basic operations. All activities use progressive schematization. The learners/children can always use visual tools if they have to. Each activity has more than one solution. The built-in constraint helps the learners to develop mathematical reasoning. In inclusive classrooms, all the learners can argue at their own level of understanding. With the Think-Aloud Method, learners become skilled at listening and building on one another's solutions. All learners are active participants in the learning process.

The general education and special education teacher candidates prepare the lesson together. During the co-teaching process, even a few words or eye contact are enough to find the common denominators and teach in harmony (the different ways of co-teaching is a separate study). The focus is on the learners' understanding and how the general education and special education teacher candidates can help the learners on their own journey and learning trajectory with the learners' individual learning needs.

The general education and special education teacher candidates make the decision about the formal and informal assessment together. Nobody is wrong in this kind of inclusive classroom. They can always refine their thinking and do it better or do it on a higher level. The general education and special education teacher candidates always pose questions that are appropriate to the learners based on their individual learning and understanding; they stretch their thinking and provide a chance to the learners to be successful and happy with their own solutions and to become thirsty for more challenges, more accomplishments and more triumphs.

If the learners cannot find the solution during the day, it is not a problem. They can go home, and if the problem was well-designed, their minds would be still working, and all of a sudden they can see the solution, and they can share it with their age group in their learning community on the

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next day. The teacher candidates have learned never to take the big 'AHA' moment from the learners but pose more questions and let them find the explanation.

Learning is a long process. We are on the way. Everybody, even the co-teaching teacher candidates can learn throughout the progress.

Now we open a window to the New York Staten Island public schools' inclusive classrooms where the two: the general education and special education teacher candidates facilitate the learning process with a well-designed progression of activities.

## 2. Method

If you want to **CHANGE** to a more productive education, we can start that during teacher education.

It is not enough to hear about it; the teachers need to practice it on their own. (Forlin, & al, 2009) "Learning by doing"

In the master's program at College of Staten Island, The City University of New York we have developed classes where special education and regular education teacher candidates can learn and student teach together. We learn how to co-teach in class, and practice it on site in Staten Island New York public schools. The following sample is from the math classes, where general education and special education students learn together in inclusive classrooms. The lesson we used was designed for inclusive classrooms.



Figure 1. Separation \* Integration \* Inclusion

**INCLUSIVE:** all children are equal citizens of the classrooms and they have two teachers: one special education and one regular education teacher. Sometimes they even have one-to-one or classroom paraprofessionals, who can provide help for the children if they have special needs, such as behavioral or other requirements.

Goal for the inclusive classrooms:

- Develop a classroom, a society, a community of learners where everyone can find their place using their best ability.
- If the children grow up together in their future neighborhood, these children in the classroom can naturally live together also as adults. If they work together in pairs and or in groups and learn to listen to each other's ideas and build on them, they can develop their own findings.

- At the end of the group work, we have the so called Math Congress, where the groups share their own findings in front of the whole class. They learn to evaluate each other's solutions, and feel free to share their own findings.

## 2.1. NUMBER SENSE

Early Childhood K-1<sup>st</sup>

In early childhood inclusive classrooms, we find that the learners/children gain a deeper understanding of **number sense**, if they play with dices and playing cards. The manipulative itself does not help. The general education and special education teacher candidates had to use a well-developed series of activities based on progressive schematization, developed by Fosnot and Dolk. (Fosnot & Dolk, 2001)



Figure 2.

## 2.2. ADDITION AND SUBTRACTION

Elementary Education 1<sup>st</sup> -2<sup>nd</sup>

Elementary education learners can learn the **basic operations of addition and subtraction** with the tool called Rekenrek. This manipulative was developed by Adrian Treffers, mathematics curriculum researcher at the Freudenthal Institute. On the top row, there are five red and five white beads; the second row is exactly the same. If the learner has to figure out what seven plus eight is with the Rekenrek:

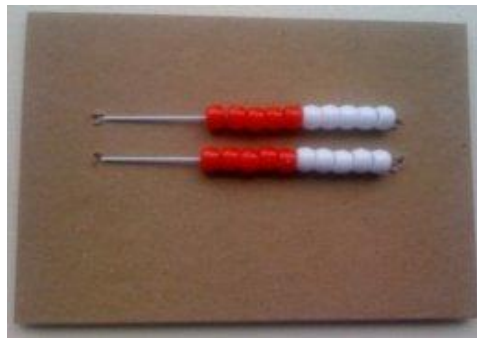


Figure 3. Rekenrek

With one push they make seven, with five red and two white beads on the top row, and on the bottom row they make eight with one push, with five red and three white beads.

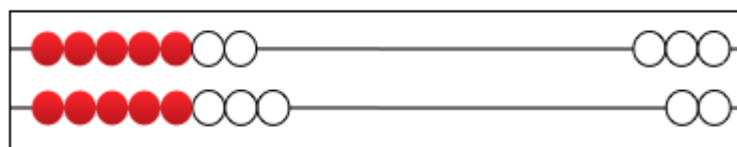


Figure 4. Seven plus eight

The learners can see five red on the top row and five red on the bottom row to be ten red beads. The only thing they have to worry about is: what two plus three white beads are. Which one is a lower level problem? The strategy is the built in five structure. The Rekenrek also developed the same manipulative for the doubling, or the doubling  $\pm 1$  strategies. In this case: seven plus seven, is the double of seven, and because the bottom row has a number that is one more, than seven, so: the double of seven is fourteen plus one more is fifteen. Or the double minus one strategy is: double of eight is sixteen, and one less is fifteen too. The special needs children use the red and white beads at the beginning, later on the learner can see the built in five structure, or the doubling, doubling plus one, or the doubling minus one strategies.

If the learners are able to see and internalize the different strategies, they can do it without manipulatives. (For example, the learners are able to do it without the Rekenrek, with the five red and five white beads on both rows. It is in their mind already.)

Using the progressive schematization, once they gained confidence with the addition and subtraction up to twenty, they can challenge each other inside groups with what is seventy plus eighty? –Can you do it in a different way? Can you reason why? After that: –What about seven hundred plus eight hundred, or –What about seven thousand plus eight thousand? (Montessori) They naturally learn to use the proper mathematical language, and every learner learns to think aloud. (Kerekes & King, 2015), (Tournaki, Young & Kerekes, 2008)

### 2.3. MULTIPLICATION

Elementary Education 3<sup>rd</sup> -4<sup>th</sup>

Well-designed activities expand the children's thinking. This series of activities were developed for children who are in inclusive classrooms. Teacher candidates field-tested these activities, and found that pictures with built in constraint especially helped the special education children to understand multiplication, and to develop multiplicative reasoning. If they have a problem, first they look at the picture and think about the possible solutions, think aloud, later they can refine their original thinking. There is more than one way to solve a problem (if the problem is well designed). Learners can appreciate each other's elegant solutions. As long as they explain their way of thinking or reasoning, they learn different solutions, and also learn to reorganize their thinking in order to help others follow it. And they also learn to pose questions to each other. They learn to work and think together.

In chapter four, there is an example of how we can help the learners understand multiplication with the built in constraint picture which was developed in the Netherlands, by the Freudenthal Institute. (Lyublinskaya & Kerekes, 2010)

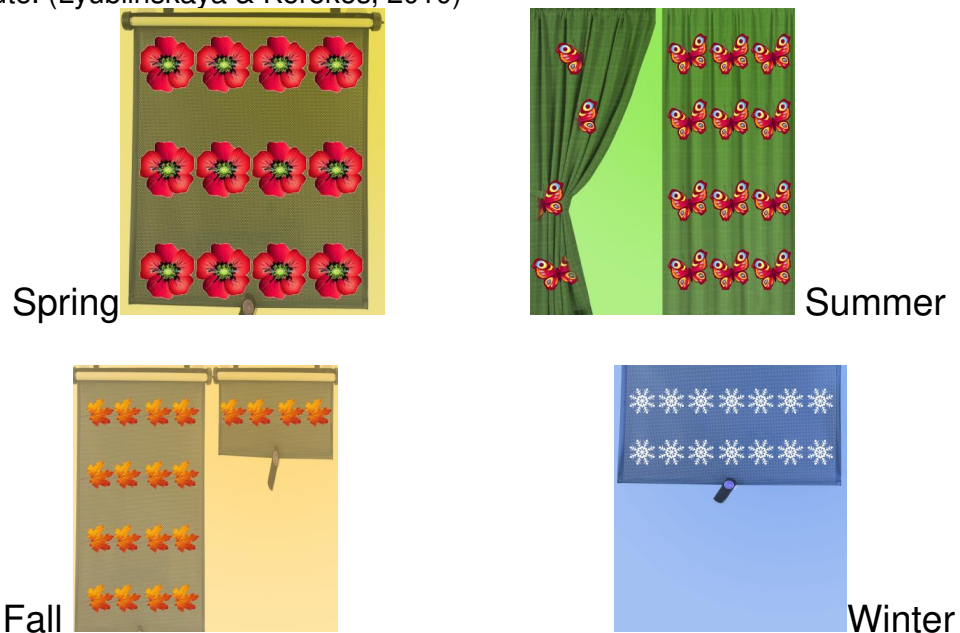


Figure 5. Four seasons

### 3. PROGRESSIVE SCHEMATIZATION:

#### 3.1. SPRING



Figure 6. Spring

Example: In the spring window they can see every single item, the flowers separately; they can touch it if necessary with their finger, or see it with their eyes.

The learner can see vertically and say  $3+3+3+3$

The learner can see horizontally too and say  $4+4+4$ , they can see as repeated addition.

Addition is linear.

$$3 + 3 + 3 + 3 +$$

The learners can see and understand how it does more.

The learner can also skip count. 3, 6, 9, 12

Or horizontally: 4, 8, 12 this is really the multiplication table

of the three and the four, developed by the learner.

The learners think they are playing with the flowers (or any other objects), **but step by step**, the learners develop their own “multiplicative reasoning”.

Multiplication itself is very hard.

-Why? Because: the learners have to think about two things at the same time.

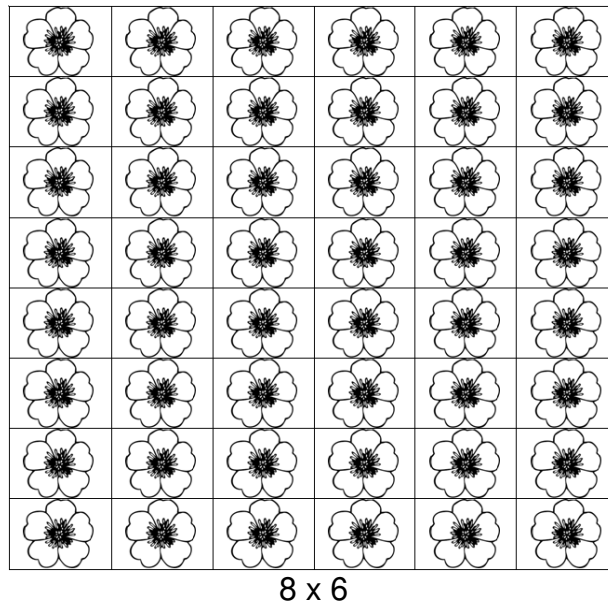
The learner can memorize the multiplication table, but they don't know why.

They have developed their “own multiplicative reasoning”, they can solve many-many problems, as opposed to each problem being a separate problem.

What is  $8 \times 6$  ? With memorization, if they remember it, they know it, if not, they don't.

But if the learners develop their multiplicative reasoning, they can find different ways to solve the problem:

Look at the pictures.



*Figure 7. Array*

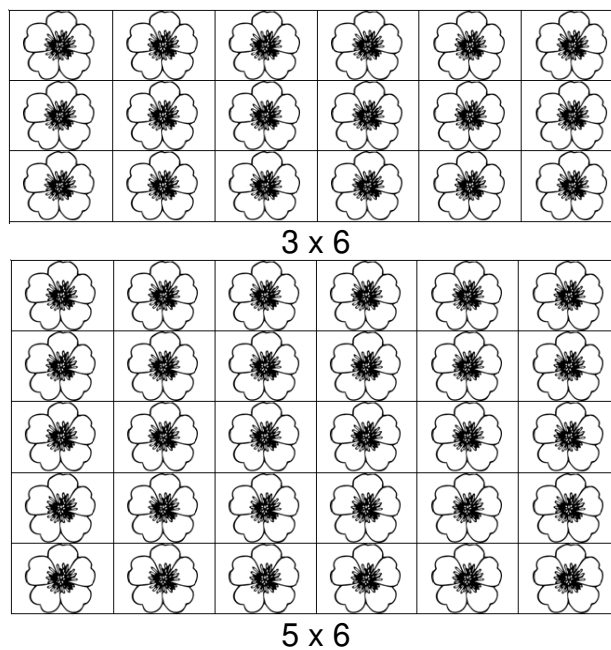
As long as the learners argue with each other:

- I can see three groups of eight  $3 \times 8 = 24$ ,
- Double it:  $6 \times 8 = 48$ .

Or: The other learners argue, if

- I do three groups of six and five groups of six
- It is still the same  $3 \times 6 = 18$  and  $5 \times 6 = 30$ , which are 48 too.

Does it follow, if I cut the array any other way? Learners learn to think aloud, argue, and prove their hypothesis, as well as gain confidence to play with the array. Learners can see and understand multiplication, the relationships and the properties of multiplication.



*Figure 8. Distributive property*

Go back to the elementary classroom. Look at the next season which is

**3.2. SUMMER:**

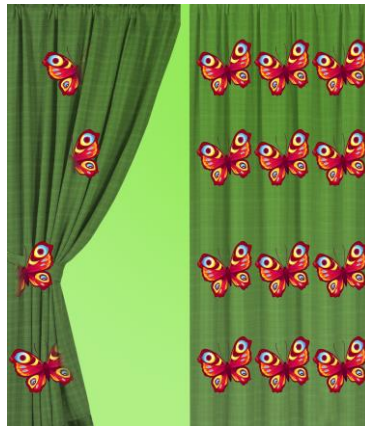


Figure 9. Summer

As we mentioned before, we use progressive schematization. The learners can only see half of the curtain, because the other half of the curtain is closed. The learners discover the doubling strategy in a real life situation.

In the inclusive classroom the two teacher candidates let the learners work in a group. They walk around the classroom to be sure that in the learning community everyone's opinion counts and everyone has the same chance to find an elegant solution. They give credit to the learners calling the solution, like Peter's strategy. Use a big chart paper that you can hang on the classroom wall with the different solutions. On the bulletin board in the hallway, parents can proudly view their children's solutions. The teacher candidates can even organize a science fair, where all the learners can proudly explain their strategies.

Any method that strengthens the learner's identity helps the learner to ask for more challenging problems, to enjoy finding the best possible solution and to get better not just in content knowledge, but also in his/her personality. The school is in the city and the city in the schools.

The inclusive classrooms provide the learners with plenty of possibilities in their own learning community.

Go back to the college classroom, where we modeled how we can learn to develop multiplicative reasoning in an inclusive classroom with two teacher candidates.

**3.3. The third season is FALL:**

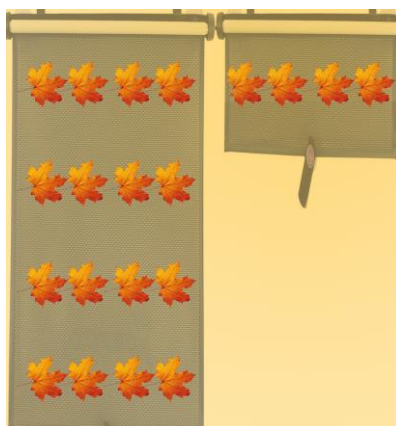


Figure 10. Fall

Progressive schematization: Now the built in constraint to the picture is not the doubling, but the doubling plus one strategy. Is this more challenging? The plus one is not really plus one; it is one more row of leaves, one more group of four leaves.

Now we are in a crucial, decisive, critical point in developing multiplicative reasoning.

#### SEE THE THINGS IN GROUPS

The built in constraint to the pictures helps the learners “see the things in groups” which is the heart of multiplicative reasoning.

### 3.4. WINTER THE KEY IS: THE PREPARATION

If the teacher candidates want to help the learners develop multiplicative reasoning, he/she has to develop pictures with built in constraints in order to stretch the learners’ thinking. (Fosnot & Dolk, 2001) The preparation before the class takes a tremendous amount of time, but it is worth it.



Figure 11. Winter

The space underneath the open shade is critical.

-How many snowflakes do we have, if we pull the curtain down? The shade in the window was not designed for two nor for three more rows of snowflakes. This design gives a chance for the learners to argue.

-I think we have 35 snowflakes. We had three rows of seven snowflakes, which is twenty-one snowflakes. I can see we have enough space for two more rows. Altogether five rows of seven snowflakes, which is thirty-five. The other group argued it is twenty-eight snowflakes all together. As long as the learners, the groups argue with each other, they learn the seven times table. They also learn to reason too.

Quote from the Irina Lyublinskaya, Judit Kerekes *Teaching Mathematics and Science in Elementary School: A technology-Based Approach*, book; chapter four: The Inclusive Classroom.

"Do your students like to dream? Imagine that after this activity you give them the following task: Think about having your own castle, house, igloo, or tent. All based in your imagination. Imagine it has four windows looking at the four seasons. Go back to your seat, and write a fairy tale. (You can use any fairy tale.) You are allowed to do whatever you want. You can finish your story at home. In the next class you can share with us your story. The writing and science will be naturally incorporated into multiplication lesson."<sup>2</sup>

We assigned our teacher candidates (general education and special education) the task of creating a fairytale. The fairytale was designed to encourage the children to solve different and increasingly more complex multiplication problems (progressive schematization). The children

<sup>2</sup> Lyublinskaya & Kerekes, 2010, pg 55

were eager to help the main character of the fairytale get free. The TC incorporated immediate feedback to the children so they knew they were on the right track in solving the problem.

- The fairytale is a tool. Like the puppet.
- That kind of learning required a lot of preparation time. The teacher candidates created their own fairytales.

### 3.5. EXAMPLE of FAIRYTALES (we blogged some of them)



Figure 12. Fairytale

1. S.L. "... Once upon a time there was a princess who lived a happy life. She loved to solve puzzles and problems and knew she was the best problem-solver in the entire kingdom. One day while the princess was in the woods, she came across a bridge over a river. As she went to walk over the bridge, a troll blocked her path. "No one gets over this bridge without solving a problem first" said the troll. "Ok." Said the princess, "I am the best problem solver in the kingdom!" "Oh yeah?" said the troll, "well in that case, you must solve four problems. ...

The princess was so happy that she was able to solve the problems. She couldn't wait to continue on her journey and tell all of her friends what just happened!"

2. L. T. The Keeper of the Seasons. "Once upon a time, in a little village a little girl was born.



Little did anyone in the village know, baby Ella was destined to become a hero. For many years the people in the village suffered from inconsistent weather patterns. The elders in the village remembered a time where there would be 4 separate seasons. Each season would come and go and people lived a very happy life. A child would hold the power to control the seasons but the task would not be easy. "

The teacher candidates' fairytales were very popular in the public schools. The learners were eager to help the main character:

- to solve problems
- to open the four windows
- to solve more challenging multiplication problems
- to use different strategies

We found that not only did the teacher candidates enjoy creating their own fairytales, but the public school children were very involved in helping the main character. They were taught as they played. At the same time **they learned many different strategies, models, big ideas to solve**

**multiplication problems with the built in constraint pictures.** At the end, their multiplicative reasoning developed, and they were able to use it in real life situations. Learners enjoyed the multiplication lessons, and **did better on multiplication tests.**

#### 4. Assessment:



Figure 13. How many flowers? How do you know it?

#### 5. Conclusions:

- In that kinds of learning environment all learners get the feeling, they are valued in the same way, equally.
- The structure of the well-designed activities offered the same possibilities of finding one or more solutions.
- If learners could not find the solutions immediately, they learned it from their fellows, whose word was closer to their own vocabulary and understanding.
- It developed a healthy and engaging learning environment, where all tried to figure out different strategies.
- The big AHA moment was greatly appreciated
- The different cultural backgrounds also predisposed their findings and reasoning.
- Success helped them look for more and/or more elegant solutions.
- The diverse learning experience opened a new vision for them.
- Different kinds of formative and informative assessment provide plenty of possibilities for the teachers to evaluate and grade the learners.
- The neighborhood involvement shaped a new school in the city.

#### 6. Discussion:

- Research conducted over a longer period of time would produce advanced conclusions which would better serve the diverse needs of the learners.
- The “two teacher model” would be more effective if the teacher candidates could observe how other special education and general education master teachers co-teach.
- Learners are more open to new learning procedures than the teacher candidates.

## 7. Results:

- As a result, children gained deeper understanding of number sense, operations and multiplicative reasoning.
- Learners were allowed to use their own speed of learning.
- Interestingly, most of the learners enjoyed, and accomplished it.
- Learners will be able to think, elaborate, and solve most real life problems on their own. Teacher candidates can teach with confidence, what they did on their own.

**Together we can make a difference:** “Great oaks from little acorns grow.”  
*English Proverb* When you believe in yourself the sky is the limit! Together we can make a difference.

**It is essential** to open the classrooms in front of our teacher candidate’s eyes, because most of them grow up in different classroom circumstances.

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